

Closing Thurs: HW 10.2

Closing Tues: HW 10.3, 11.1&2(part 1)

10.3 Curve Sketching Summary Local and Global Max/Min

Entry Task: Consider

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$

1. Find all critical values.
2. Plug the critical values into the 2nd derivative. What can you conclude?
3. Draw the 1st and 2nd deriv. analysis number lines.

$$\boxed{1} \quad f'(x) = 3x^2 - 9x - 12 \stackrel{?}{=} 0$$

$$3(x^2 - 3x - 4) \stackrel{?}{=} 0$$

$$3(x - 4)(x + 1) = 0$$

OR
USE
QUAD.
FORMULA

$$\boxed{x = -1 \text{ or } x = 4}$$

$$\boxed{2} \quad f''(x) = 6x - 9$$

$$\text{AT } x = -1, \quad f''(-1) = 6(-1) - 9 = -15$$

$$\text{So } f'(-1) = 0 \quad \text{HORIZ. TANGENT?}$$

$$f''(-1) < 0 \quad \text{CONCAVE DOWN}$$

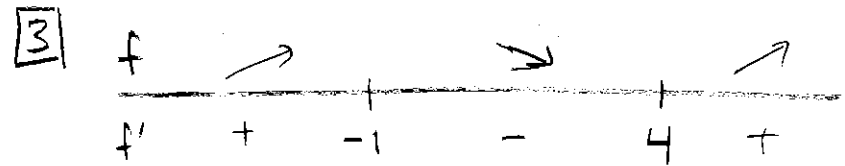
LOCAL
MAX!

$$\text{AT } x = 4, \quad f''(4) = 6(4) - 9 = 15$$

$$\text{So } f'(4) = 0 \quad \text{HORIZ. TANGENT?}$$

$$f''(4) > 0 \quad \text{CONCAVE UP}$$

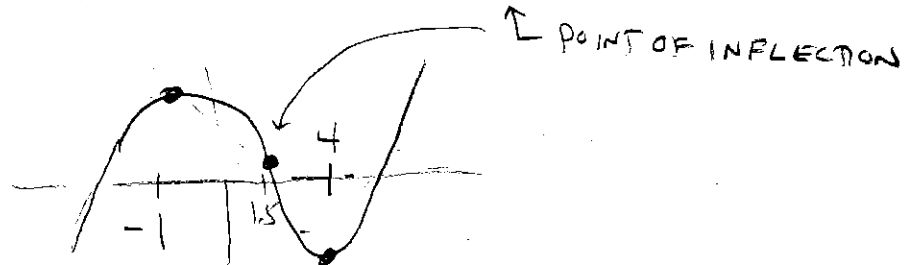
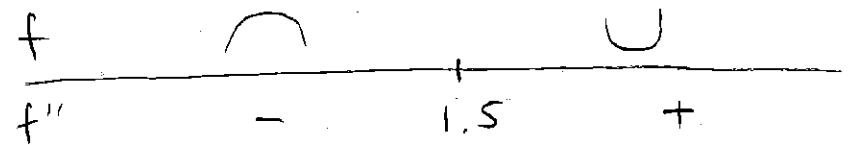
LOCAL
MIN!



$$f'(x) = 3(x - 4)(x + 1)$$

$$f''(x) = 6x - 9 \stackrel{?}{=} 0 \Rightarrow 6x = 9$$

$$\Rightarrow x = \frac{9}{6} = \frac{3}{2} = 1.5$$



Summary

For any question about “**increasing, decreasing, local max/min**”, identify the function in question, $y = f(x)$, then:

Step 1: Solve $f'(x) = 0$

Step 2:

(option 1): 1st Deriv. Test

Draw 1st deriv. analysis number line.

Make appropriate conclusions.

(option 2): 2nd Deriv. Test

Plug critical numbers into 2nd deriv.

• $f'(a) = 0, f''(a) > 0 \Rightarrow$ local min \cup

• $f'(a) = 0, f''(a) < 0 \Rightarrow$ local max \cap

For any question about “**concave up/down or inflection points**”, identify the function in question, $y = f(x)$, then:

Step 1: Solve $f''(x) = 0$

Step 2:

Draw 2st deriv. analysis number line.

Make appropriate conclusions.

Global Max/Min:

Given $y = f(x)$ and an interval $a \leq x \leq b$

The **global maximum** (or *absolute max*) of $f(x)$ on the interval is the highest overall y -value on that interval.

The **global minimum** (or *absolute min*) of $f(x)$ on the interval is the lowest overall y -value on that interval.

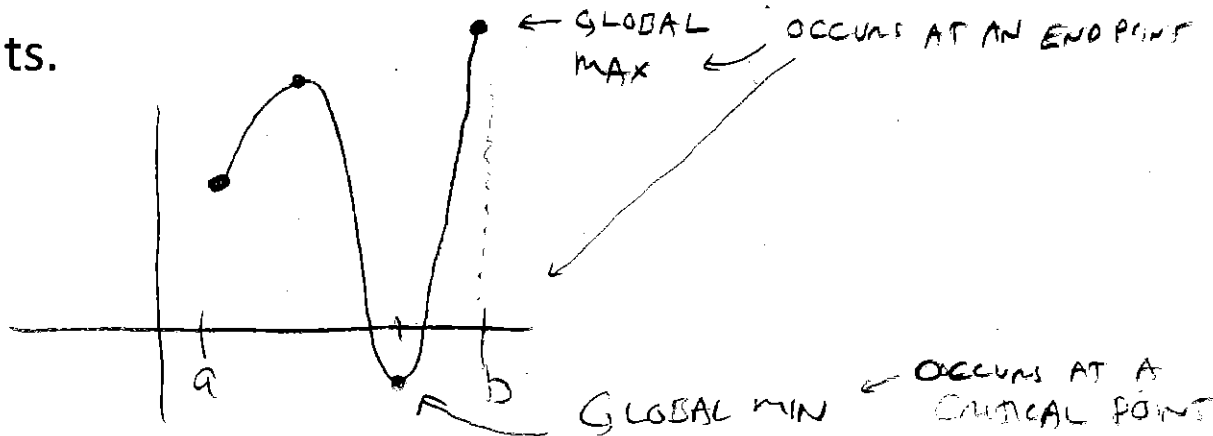
Key (awesome) Observation (Extreme Value Thm)

The global max/min can only occur at:

a. critical values

OR

b. endpoints.



For any question about “**global max/min**”, identify the function and interval in question, $y = f(x)$ and $a \leq x \leq b$ then:

Step 1: Solve $f'(x) = 0$

Step 2:

- Plug the critical values into the original function.
- Plug the endpoints into the original function.

At the end of step 2:

The biggest output is the global max.

The smallest output is the global min.

Example (same function from entry task):

On the interval $-2 \leq x \leq 10$, find the global max and min of

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$

$$f'(x) = 3x^2 - 9x - 12 \stackrel{?}{=} 0 \quad \left. \begin{array}{l} \text{already} \\ \text{did} \\ \text{(see page 1)} \end{array} \right\}$$

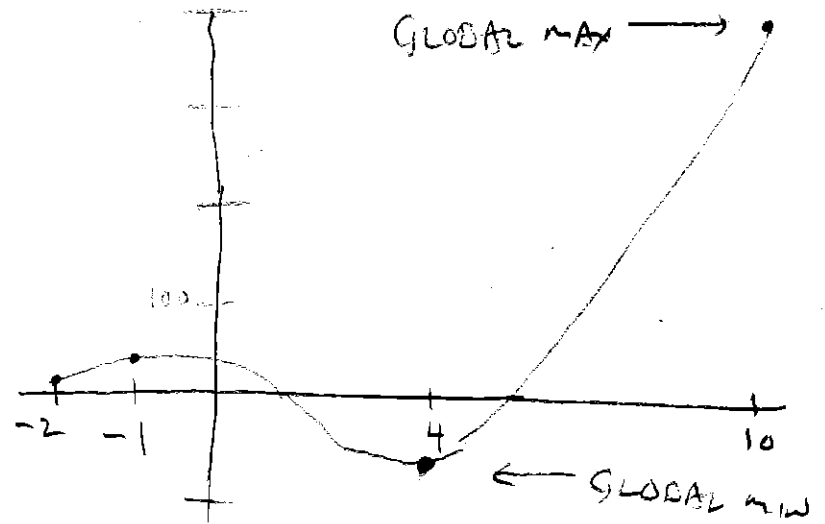
$x = -1, x = 4$

$$f(-1) = (-1)^3 - \frac{9}{2}(-1)^2 - 12(-1) + 10 = 16.5$$

$$f(4) = (4)^3 - \frac{9}{2}(4)^2 - 12(4) + 10 = -46 \quad \leftarrow \text{GLOBAL MIN}$$

$$f(-2) = (-2)^3 - \frac{9}{2}(-2)^2 - 12(-2) + 10 = 8$$

$$f(10) = (10)^3 - \frac{9}{2}(10)^2 - 12(10) + 10 = 440 \quad \leftarrow \text{GLOBAL MAX}$$



Example (from HW 10.3/10):

The total revenue (in thousand dollars) for selling q thousand Framits is given by

$$TR(q) = \frac{1}{6}q^4 - \frac{31}{6}q^3 + 55q^2 + 200q$$

Part (c) Find the global max and global min of marginal revenue over the interval $q = 0$ to $q = 12$.

$$MR(q) = \frac{4}{6}q^3 - \frac{31}{6}q^2 + 110q + 200$$

$$MR(q) = \frac{2}{3}q^3 - \frac{31}{2}q^2 + 110q + 200 \quad \leftarrow \text{QUESTION IS ABOUT MR!!!}$$

$$MR'(q) = 2q^2 - 31q + 110 \stackrel{?}{=} 0$$

$$q = \frac{31 \pm \sqrt{(-31)^2 - 4(2)(110)}}{2(2)} = \frac{31 \pm \sqrt{81}}{4}$$

$$= \frac{31 \pm 9}{4} = \rightarrow \begin{array}{l} 40/4 = 10 \\ \text{or} \\ 22/4 = 11/2 = 5.5 \end{array}$$

$$MR(5.5) = \frac{2}{3}(5.5)^3 - \frac{31}{2}(5.5)^2 + 110(5.5) + 200 = 447.041\bar{6}$$

$$MR(10) = \frac{2}{3}(10)^3 - \frac{31}{2}(10)^2 + 110(10) + 200 = 416.\bar{6}$$

$$MR(0) = \frac{2}{3}(0)^3 - \frac{31}{2}(0)^2 + 110(0) + 200 = 200$$

$$MR(12) = \frac{2}{3}(12)^3 - \frac{31}{2}(12)^2 + 110(12) + 200 = 440$$

$$\text{GLOBAL MAX} = 447.041\bar{6}$$

$$\text{GLOBAL MIN} = 200$$

Example: (like the last problem in HW)

Given $g(x) = \frac{1}{4}x^2 - 4x + 25$ and

$$S(x) = \frac{g(x)}{x}$$

If x is between 1 and 20, what is the smallest possible value of $S(x)$?

$$S(x) = \frac{\frac{1}{4}x^2 - 4x + 25}{x} = \frac{1}{4}x - \frac{4x}{x} + \frac{25}{x}$$

$$S(x) = \frac{1}{4}x - 4 + 25x^{-1}$$

$$S'(x) = \frac{1}{4} - 25x^{-2} \stackrel{?}{=} 0$$

$$4x^2 \left(\frac{1}{4} - \frac{25}{x^2} \stackrel{?}{=} 0 \right)$$

$$x^2 - 100 \stackrel{?}{=} 0$$

$$\Rightarrow x^2 = 100 \Rightarrow x = \pm 10$$

ONLY ONE BETWEEN
1 AND 20

$$S(1) = \frac{1}{4}(1) - 4 + \frac{25}{1} = 21.25$$

$$S(10) = \frac{1}{4}(10) - 4 + \frac{25}{10} = 1$$

$$S(20) = \frac{1}{4}(20) - 4 + \frac{25}{20} = 2.25$$

GLOBAL MIN = 1

Example: (like problems 5-9 of HW)

Given the monthly average cost and price for producing and selling q items:

$$AC(q) = \frac{36000}{q} + 100 + q$$

$$p = 1700$$

If production is limited to 400 items per month, what quantity maximizes profit?

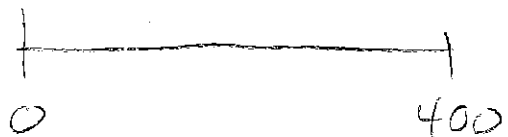
$$TC(q) = q \cdot AC(q) = 36000 + 100q + q^2$$

$$TR(q) = p \cdot q = 1700q$$

$$P(q) = TR(q) - TC(q) \\ = (1700q) - (36000 + 100q + q^2)$$

$$P(q) = 1600q - 36000 - q^2$$

WHAT IS GLOBAL MAX IF $0 \leq q \leq 400$?



$$P'(q) = 1600 - 2q \stackrel{?}{=} 0$$

$$1600 = 2q$$

$$800 = q$$

↑
OUTSIDE OF INTERVAL!!!

$$P(0) = 1600(0) - 36000 - (0)^2 = -36000$$

$$P(400) = 1600(400) - 36000 - (400)^2 \\ = 444,000 \leftarrow \text{MAX PROFIT}$$

$$q = 400$$

For $0 \leq q \leq 400$

NOTE: IF PRODUCTION
COULD BE HIGHER,
THE MAX PROFIT
WOULD OCCUR AT 800
 $P(800) = 604,000$